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SPECIAL REPORT

A Multicriteria Decisionmaking Framework For the Evaluation of Forest Road Investment Projects

Luiz Flávio Autran Monteiro Gomes

Prepared for the U.S. Forest Service
Transportation Analysis Group
Under Contract No. GW 25234

On February 1, 1976 the University of California's systemwide Institute of Transportation and Traffic Engineering was formally renamed THE INSTITUTE OF TRANSPORTATION STUDIES. Our address and telephone number on the Berkeley campus and on the Irvine campus remains the same.

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THE INSTITUTE OF TRANSPORTATION AND TRAFFIC ENGINEERING
UNIVERSITY OF CALIFORNIA

Institute of Transportation
and Traffic Engineering
Special Report

A MULTICRITERIA DECISIONMAKING FRAMEWORK FOR THE EVALUATION OF FOREST ROAD INVESTMENT PROJECTS

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University of California
Berkeley, February 1976

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Luiz Flávio Autran Monteiro Gomes

Abstract

A conceptual and operational evaluation framework that takes into account quantifiable as well as non-quantifiable forest road investment criteria is presented. The framework allows putting together existing functional analysis methodologies and value systems of different participants in the evaluation process. The framework is oriented toward social choice.

A "projects' valuation matrix" is defined as a matrix whose columns are vectors of measures of partial utility of projects. Since evaluation criteria can be either quantitative (e.g. discounted costs) or qualitative (e.g. scenic value), some columns of the projects' valuation matrix will correspond to quantitative criteria while other columns will correspond to qualitative criteria.

While measures of partial utility of projects given quantitative criteria can be directly computed, qualitative criteria require value judgements from participants in the process. These value judgements are expressed by forming matrices of direct pairwise comparisons among projects. Each matrix of direct pairwise comparisons is constructed for a qualitative criterion.

An algorithm to develop measures of indirect pairwise comparisons is introduced. By its application it is possible to determine matrices of total pairwise comparisons among projects and compute the vectors of measures of partial utility.

A reference criterion is identified by going through the process of computing measures of total pairwise comparisons among criteria and determining a vector of weights for all criteria. The information contained in the projects' valuation matrix is then synthesized by using trade-off rates among criteria to translate all criteria into the reference criterion. A matrix of indices of relative dominance for pairs of projects is then formed. Indices of overall dominance or composite weights for all projects are then computed by summing over the rows of the matrix of indices of relative dominance and by normalization.

A heuristic procedure to facilitate the application of the framework is suggested. The procedure is applied to examples and it is found that the loss of information involved is not significant for practical purposes.

Although the algorithm used to form a projects' valuation matrix was designed to deal with independent projects, a method to handle contingent investment projects is suggested and applied to situations which are most likely to occur in practice. A numerical example is given.

The framework was found suitable for practical applications and results from its application seem consistent with the preference structures of decision makers. The whole framework can be implemented as a conversational computer program so as to become a man-computer interactive evaluation tool useful for transportation analysts.

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Glossary

Algorithm: a set of mathematical computations

Anchoring effects: a type of bias that occurs in repetitive experiments; with each succeeding presentation of the experiment, the observer's judgements are modified so that, in a very short time, judgements are largely made relative to the particular range of stimuli presented in the experiment.

Cardinal scale: a scale in which a weight is assigned to every object.

Choice: the political process by which different participants look at all alternatives as ranked by the framework, and change ranks according to their particular viewpoints.

Community value analysis: an analysis of value structures and decisions made by all participants in the evaluation process.

Composite weights (or scores): weights of projects that take into account the relative importance of each project with respect to every objective as well as the relative importance of every objective.

Criterion: a mathematical entity that can either be an ordinal variable, when the relative importance of each project is given by its rank, or a cardinal variable, when a weight is assigned to each project.

Contingency factor: a multiplicative factor used, whenever projects are interrelated, to correct the measures of partial utility that are obtained by making the (false) assumption that projects are mutually exclusive.

DELPHI method: a method to achieve consensus based upon individual order structures; those orders are not aggregated, but they are progressively modified.

Digraph: a graph whose arcs are directed.

Dominance: a relation between two objects in which the behavior of one object depends on the behavior of the other object.

Functional analysis: planning analysis oriented towards specific functions, such as minimizing the cost of timber haulage, optimizing the location of fire stations, etc.

Heuristic: valuable for empirical work but incapable of proof.

Hierarchically structured system: a system organized in layers in which the behavior of a given layer depends on the behavior of the layer which is immediately above.

Hierarchy: a level of importance or dominance in a network.

Indifference curve: the locus of all combinations of criteria which create the same amount of satisfaction.

Information content: amount of data, available in a matrix of pairwise comparisons, that helps someone to make a decision in terms of ranking alternatives.

Input-output matrix: a matrix that displays, for a given economy, the inputs and outputs of each industry as well as their interrelationships.

Investment project: any planned improvement in associated links of a network, implying both capital and operational expenditures.

Linear programming: a mathematical technique to optimize an objective function subject to constraint equations, given that the objective function and the constraint equations are linear.

Man-machine interaction: a form of interaction between an analyst and the computer in which the analyst looks at the results of computations and changes the values of some of the variables in order to obtain new numerical results; changes are introduced and new

numerical results are obtained until the analyst is satisfied with the computed values.

Multiple objective decision making: a branch of decision theory which deals with the process of making decisions under various and usually conflicting objectives.

Ordinal scale: a scale in which the relative importance of a given object is given by its rank.

Pairwise comparisons: a pairwise comparison between two objects is a number that reflects how important one object is compared to the other, given a certain objective.

Partial utility: the utility of one project with respect to a particular criterion.

Projects' valuation matrix: a matrix in which each row is associated with one project and each column is associated with one criterion; any entry of the matrix is a measure of partial utility of a given project with respect to a certain criterion.

Publicly accountable process: a process by which alternatives are ranked and displayed in a way such that different interest groups can easily perceive how the ranks affect their interests.

Response: reaction of an observer to a stimulus.

Similarity matrix: a matrix whose entries are measures of proximity between projects, that is, the entries are the absolute differences between absolute scores tentatively assigned to the projects.

Stimulus: any object or event that causes an observer to react in some way.

Synthesis: a process by which pairwise comparisons among criteria are made, a reference criterion is identified, and the measures of partial utility displayed in the projects' valuation matrix are

aggregated by translating all criteria in terms of the reference criterion; vectors of composite weights are then computed and the overall priority of each project is determined.

Utility: a measure of satisfaction.

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Chapter I

Introduction

I.1 The Need for a Transportation-Resource Analysis Interface

The evaluation of forest road investment projects generally involves different interest groups and Forest Service personnel faced with multiple objectives. The role played by technical analysis is to go from the simultaneous observance of these objectives to the organization and display of information which will serve as input to choice.

The connection between decision making processes in road and resource planning in a national forest has recently been presented in a didatic way by Sirmon and Shur (1). As those authors point out, the multifunctional characteristics of road investment decisions greatly increase the complexity of decision making, and there is need for a systematic approach to road investment decision making that simultaneously takes into account the many different criteria which are used to evaluate investment projects. Moreover, road investment decision making should be designed to facilitate involvement of the public in the formulation, comparison, and selection of alternatives.

A number of functional analysis methodologies to support systematic planning exist at different stages of development, and have been reviewed by Kirby et al. (2). Some of these methodologies, when fully developed, will serve as inputs to others (e.g., results from the Timber Transport Model can be inputs to Timber RAM and vice-versa).*

*The Timber Transport Model and Timber RAM are mathematical programming techniques for determining, respectively, the transport strategy and timber resource management strategy that maximize economic present value. While familiarity with such methods is helpful for understanding the application environment of the present evaluation-decision making framework, it is not essential, since the framework can be understood independently of its area of application described here.

The essential objective of all of them, however, when viewed from the angle of road investment decision making, is to help choose among alternative network investment strategies for road construction and maintenance.

The multicriteria decision making framework developed herein consists of a conceptual and operational device to integrate different investment criteria -- some of those making use of the above-mentioned functional analysis methodologies, others reflecting value judgements of Forest Service personnel representing professional disciplines involved in the resource management process, and still others generated through public involvement in the investment decision making process.

I.2 Relationship to Previous Work

The problem of assigning weights reflecting individual preferences to projects in a network, given a multiplicity of evaluation criteria, has been approached by a few authors (3,4,8). Existing procedures, however, do not provide treatment of interdependencies among projects, nor are they oriented toward choice.

Our framework utilizes the idea of making pairwise comparisons among projects, which is embedded in Saaty's method (8). However, we introduce modifications not only in the way to compute the weights, but also in the mathematics involved.

Our idea of using a reference criterion evolved from Bertier and Montgolfier's work (4), although our method to select that reference criterion relies more on technical analysis than these two authors'. Instead of choosing the reference criterion directly, as these two authors suggest, we use as input to its selection a matrix of trade-off rates between criteria, in which we assume those rates to be mutually

independent and independent of the criteria's utilities.

The framework as a whole is suitable for implementation as a man-computer interactive evaluation device, and promotes the openness of project evaluation to participation by different interest groups. In this sense our framework consists of a departure from the traditional multicriteria analysis, to the extent that it treats evaluation as a publicly accountable process.

Chapter II

The Analytical Framework for the Evaluation of Forest Road Investment Projects

II.1 Multicriteria Approach to Forest Road Project Evaluation

The evaluation of forest road investment projects must be understood as a process by which each project is analyzed to determine how well it matches a set of specified objectives from the points of view of various interest groups. This concept of evaluation, therefore, involves six major steps:

--identification of goals and objectives that serve to establish the evaluation criteria;

--formal definition of the set of basic viewpoints and criteria on which to base the comparison of projects;

--definition of measures utilized to characterize the value of consequences of different projects;

--evaluation of the performance of each project in relation to each criterion;

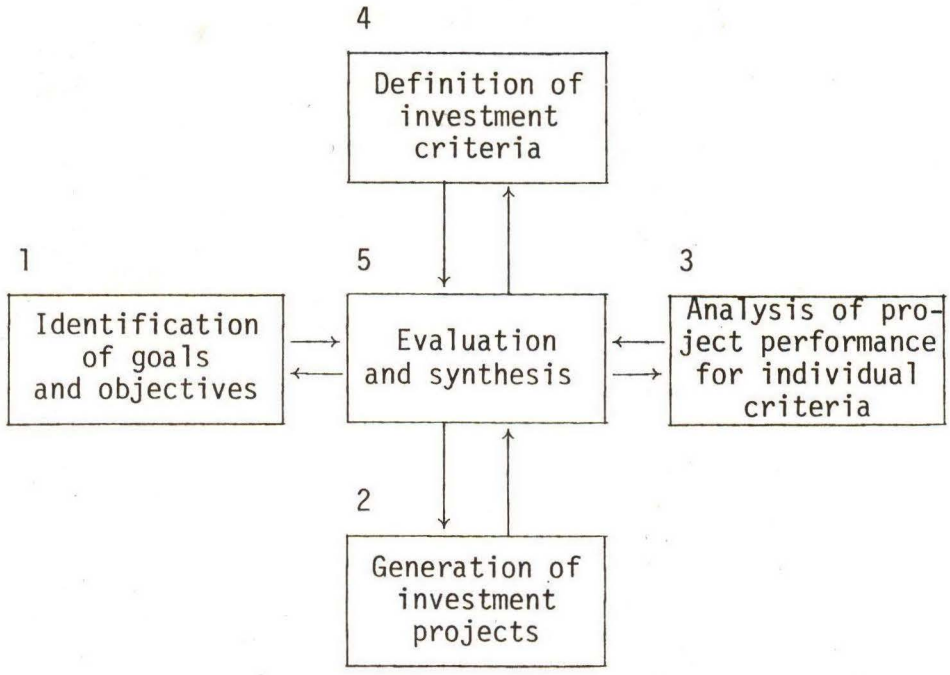
--selection of a method to aggregate the performance of all projects in relation to all criteria, thus obtaining a synthesis;

--development of a paticipatory organizational structure to allow finding not just a "best" solution, but several best solutions as seen from different points of view.

Evaluation, therefore, should lead us to different scenarios and choice should be exercised through the political process.

The diagram below illustrates the way the evaluation process is viewed. We observe that, as opposed to the conceptual structure reflected in that diagram, the linear sequence of tasks 1-2-3-4-5 has

characterized the traditional approach to road investment planning.



According to this view of the process, evaluation depends on making complete information available to participants and investment criteria are introduced or removed according to the reaction of participants. In this sense the forest road decision making process is seen as an open process.

On the other hand, transportation analysts have agreed that the traditional criteria of cost-benefit, cost-effectiveness, and effectiveness analysis are inadequate to handle resource-oriented effects such as damage to or enhancement of wildlife and watershed, access to game species, access to archaeological and historical resources, scenic and aesthetic quality of roads, privacy, safety, accessibility to fire control, etc., even if one considers improvements in analysis tools which are likely to occur in the future.

A multicriteria approach to the forest road decision making process

takes into account the value systems of different participants in the process; each of several best solutions generated reflects a "best compromise." Through this approach, economic analysis relaxes its formal exactitude but profits in realism.

We address ourselves to the problem of evaluating the performance of each project in relation to each criterion and aggregating this information through a synthesis. Important issues related to the selection of a set of basic investment criteria are discussed by Sirmon and Suhr (1) as well as Lachêne and Morice (3).

Before presenting the analytical framework of the proposed decision making approach, it is useful to explain the conceptual distinction between "investment criterion" as this term is used by Sirmon and Suhr (1), and "criterion" as this word is utilized in the next sections. By an investment criterion, these two authors mean any aspect of reality that participants in the evaluation process may consider as relevant while selecting investment projects. Thus, for example, transportation and resource systems analysis can be one investment criterion if participants agree that investment decisions should be based on combined analysis of alternative resource and transportation systems, among other considerations.

By a criterion, on the other hand, we mean a variable expressing an investment objective or point of view, as that term is defined by Bertier and Montgolfier (4). According to these two definitions, therefore, an investment criterion of Sirmon and Suhr can be expressed by one or more of our criteria. To avoid any confusion in the use of these two terms, Sirmon and Suhr's use of the expression "investment criterion" is replaced by "point of view" from now in this paper.

A criterion is a mathematical entity and can either be an ordinal

variable, when the relative importance of each project is given by its rank, or a cardinal variable, when a weight is assigned to each project. In general, ordinal criteria express qualitative points of view and cardinal express quantitative.

As compared with cardinal criteria, however, ordinal criteria lead to a certain loss of information. In the case of ordinal criteria, the first rank is assigned to the project showing best performance and the other project ranks follow in order of decreasing performance. In the case of cardinal criteria, on the other hand, a scale of weights is selected (e.g., from 0 to 1) and the highest weight is assigned to the project showing best performance, while the other project weights are proportionally determined with performances expressed in specific units.

In this framework, it is advantageous to establish cardinal scales for all criteria, even those that are qualitative. Fortunately, it is possible to establish a correspondence between a scale of relative ranks and a cardinal scale. As is shown below, this representation of qualitative criteria by cardinal scales is a major feature of the framework.

II.2 The Concepts of "Projects' Valuation Matrix" and "Synthesis"

We look at the forest road investment decision making process as a two-level hierarchically structured system, in which the following elements are identified according to the terminology used by Mesarovic (5):

--overall process: the road investment decision making process;

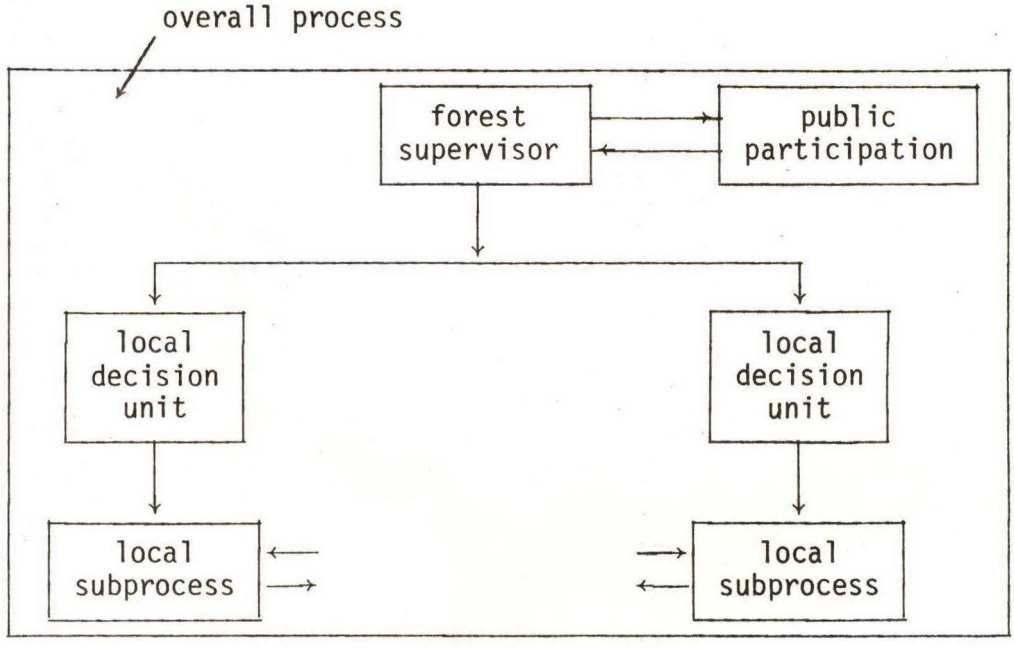
--local subprocesses: the processes by which investment projects are weighted or ranked according to different criteria and new projects are

generated according to the reactions of participants in the overall process;

--first-level decision units: Forest Service personnel representing the professional disciplines and points of view involved in the process as well as the public;

--coordinator: forest supervisor, who also serves as catalyst in the community value analysis.

The structure of the overall process is illustrated in the diagram below:

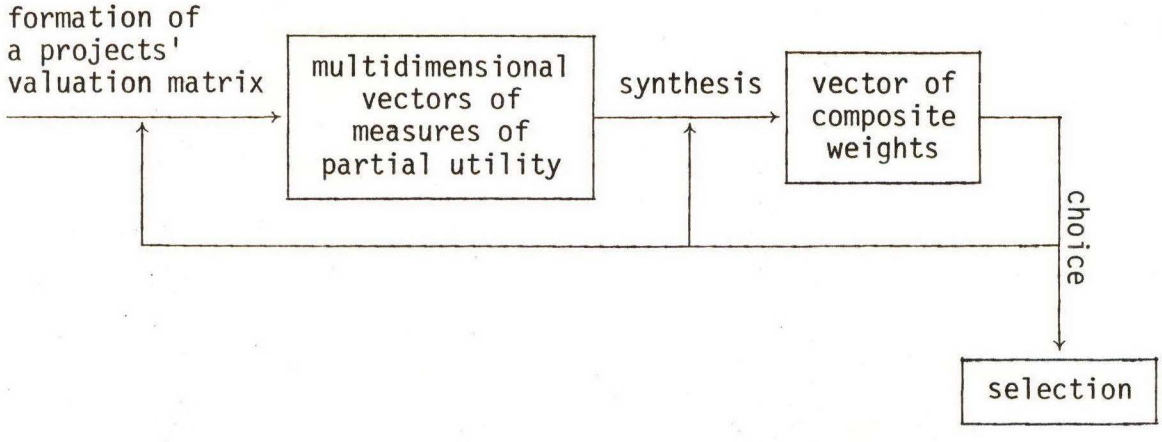


Through this view of the decision making process, local decision units compare all investment projects according to every criterion and produce as many order of project priorities as the number of criteria. We see later that, for each criterion, we can always assign numbers from 0 to 1 to every investment project. In other words, even if we are trying to order projects given a qualitative criterion (e.g., scenic and aesthetic quality of roads), it is possible to rank each project

on a numerical scale, i.e., each criterion is a cardinal variable.

The forest supervisor, on the other hand, compares all criteria and establishes a corresponding order of priorities. The synthesis problem then consists of combining the many different vectors of numerical values from 0 to 1 (each vector corresponding to a different criterion) with another similar vector of values (each of those values corresponding to a particular criterion).

The relationships among the major elements of the framework are summarized in the diagram below, where the feedbacks are required to reach political consensus:



If, at any time during the evaluation process, we have n investment projects and m evaluation criteria under consideration, it is possible to form the projects' valuation matrix shown below, in which each column is a vector of project weights:

Investment project number	Criterion number					
	1	2	...	c	...	m
1	w_{11}	w_{12}	...	w_{1c}	...	w_{1m}
2	w_{21}	w_{22}	...	w_{2c}	...	w_{2m}
⋮	⋮	⋮		⋮		⋮
i	w_{i1}	w_{i2}	...	w_{ic}	...	w_{im}
⋮	⋮	⋮		⋮		⋮
n	w_{n1}	w_{n2}	...	w_{nc}	...	w_{nm}

In this matrix, as each weight reflects the performance of a given project with respect to a particular criterion, we can also call these weights "measures of partial performance" or "measures of partial utility." In that matrix, the generic value w_{ic} $(0 \leq w_{ic} \leq 1; \sum_{i=1}^{n} w_{ic} = 1)$ represents the utility of project i with respect to criterion c.

Measures of partial utility for quantifiable criteria can be directly computed (e.g., discounted present values can be computed and then normalized). For non-quantifiable criteria, however, value judgements from decision makers can be translated into matrices of direct pairwise comparisons among projects. An algorithm is then applied to determine measures of indirect pairwise comparisons among projects and to compute measures of total pairwise comparisons. The vector of cardinal measures of partial utility for every non-quantifiable criterion can be determined from the matrix of total pairwise comparisons among criteria, as it will be shown in the next chapter.

Once a projects' valuation matrix is formed with its columns representing vectors of measures of partial utility for both quantifiable

and non-quantifiable criteria, the information contained in that matrix is synthesized without significant loss of information. This synthesis provides decision makers with a minimum amount of information for choice.

The feedbacks shown in the above diagram are generated through the process of achieving political consensus. In other words, results from a first application of the framework are presented to different interest groups. In this political arena, vectors of composite weights are changed by either changing the relative importance of criteria or measures of pairwise comparisons among projects or both. Those changes then generate new vectors of composite weights and those new vectors are again presented in the political arena until a consensus is achieved.

Chapter III is devoted to the presentation of detailed procedures to form a projects' valuation matrix as well as to synthesize the information contained in it.

II.3 Chapter Summary

In this chapter we present the conceptual characteristics of our multicriteria approach to forest road project evaluation. A choice-oriented view of evaluation is then introduced. The inadequacy of the linear evaluation and decision making process is mentioned for dealing with multi-functional, resource-oriented planning requirements.

Evaluation is defined within a participatory scheme. The concept of a criterion is explained in the context of the framework. The usefulness of ordinal and cardinal criteria are compared, and the need to utilize only cardinal criteria is explained.

After presenting the hierarchical structure of the framework, the concepts of projects' valuation matrix and synthesis are introduced. The basic distinction between the ways to obtain measures of partial

utility for quantifiable as well as for non-quantifiable criteria is explained.

Feedbacks generated through the process of achieving political consensus are mentioned as important elements of the framework.

Chapter III

Analysis of Mutually Exclusive Investment Projects

III.1 Total Pairwise Comparisons of Projects

Impacts of projects can fall into the following categories:

(i) direct impact of one project on another project,

(ii) indirect impact of one project on another project,

(iii) direct impact of one project on one objective (or criterion),

and (iv) indirect impact of one project on one objective (or criterion).

Any two projects are mutually exclusive whenever the utility of one approaches zero if the other project is adopted and vice-versa. In this case we cannot even talk in terms of the impact (either direct or indirect) of one project on another, because the two projects cannot be considered simultaneously as components of the same network. Categories (i) and (ii), therefore, are only dealt with in the context of contingent projects presented in Chapter V.

To deal with direct and indirect impacts of any project on a given objective, we start by concentrating our attention on criteria that express qualitative points of view, i.e., criteria that depend upon value judgements. This is the case, for example, of scenic and aesthetic quality of roads; no quantitative analysis technique is deemed acceptable to determine measures of performance of projects expressed in units specific to that criterion.

For any criterion c of this type, we can form a matrix A_c of pairwise comparisons among projects. If a_{ij} denotes an entry of such a matrix, we have:

a_{ij} = an estimate of the relative contribution that project i makes toward the fulfillment of the objective represented by the given criterion, as compared with the contribution made by project j.

For example, if the criterion is privacy, a_{ij} is an estimate of the contribution that project i makes to the maximization of privacy as compared with the contribution made by project j; that is, if i is about half as effective as j, $a_{ij} = .5$.

As such an estimate reflects the values of a local decision unit, an intensity scale of values must be constructed. If, for example, we choose a scale from 0 to 10, each element of A_c must satisfy the following properties:

(a) $0 \leq a_{ij} \leq 10$,

(b) $a_{ij} = \begin{cases} \frac{1}{a_{ji}} & \text{if } a_{ji} > 0 \\ 0 & \text{if } a_{ji} = 0 \end{cases}$

If both a_{ij} and a_{ji} equal zero we can say that projects i and j are non-comparable (or beyond quantifiable comparison).

The indirect impact of one project on a given objective can be assessed via either other projects or other objectives. For example, assume that, in terms of scenic value, project i is x times more important than project ℓ, and that project ℓ itself is y times more important than project j. Therefore, project i is xy times more important than project j, via project ℓ. We can then say that the intensity of the indirect interaction between projects i and j, via project ℓ, is equal to xy. Although indirect interactions are not as important as direct interactions, the inclusion of the former in the analysis represents an increase of information.

While determining indirect impacts of projects on objectives via other objectives depends upon the way we synthesize information contained in the projects' valuation matrix, indirect impacts of projects on objectives via other projects can be determined by making pairwise comparisons between projects.

In fact, although the entries of the matrix A_c are determined by direct comparisons between projects, the information can be manipulated to obtain comparisons at different levels. What is needed is a procedure that uses the A_c matrix to compute a vector of weights for all n projects, such that direct as well as indirect comparisons between projects are taken into account.

To obtain such a procedure, we start by mapping our set of n investment projects onto a digraph whose arcs and nodes have the following properties:

(a) each node i represents the mapping of investment project i;

(b) to each arc (i,j) we associate the number a_{ij}, named the intensity of the arc and obtained from the A_c matrix. Arcs are only associated with pairs of projects that are comparable.

The problem of determining total -- direct and indirect -- interactions among objects mapped onto a digraph is generally approached by taking successive powers of the original matrix A_c and following either one of the three procedures below:

(a) Determine an accessibility matrix $T = \{t_{ij}\}$, where t_{ij} is the sum of the corresponding elements of each matrix $A_c, A_c^2, \ldots, A_c^D$, D being equal to the diameter of the graph.* This approach is explained by Taafe and Gauthier (6).

(b) As in (a), but multiply each matrix $A_c, A_c^2, \ldots, A_c^D$, by powers

*For footnote, see page 16.

of a scalar s between 0 and 1 to reflect that high-powered matrices are of diminishing importance in determining graph accessibility. This procedure is suggested by Garrison (7).

(c) Determine a list of weights by taking the limiting values of the row sums of A_c^{λ}, the λ-th power of A_c. As we normalize the row sums by dividing each of them by the sum of all row sums, each number so obtained serves as an index of importance of each object with respect to all other objects. This is the method introduced by Saaty (8).

Procedures (a) and (c) introduce redundancies in the analysis of total interactions among objects, because higher-powered measures of interaction tend to reflect lower-level measures. Approach (b) consists of an attempt to minimize these redundancies, although no guidelines exist for choosing a value for s.

All three approaches, however, introduce another type of redundancy, i.e., the existence of a loop at each node causes double and multiple counting.

It is possible to eliminate this last type of redundancy by redefining the operations of matrix multiplication and matrix summation. If a_{ij} designates a generic element of A_c, ${}^{2}a_{ij}$ a generic element of A_c^2, and ${}^{D}a_{ij}$ a generic element of A_c^D, these redundancies are eliminated and reciprocity is guaranteed at all levels if we have the following:

---

*Footnote to page 15: Although it is mathematically possible to compute matrices of powers higher than the diameter of the corresponding graph, a graph with n nodes can have no sequence of connections greater than $n-1$ without establishing a path that passes by at least one node more than once. In other words, any path of length greater than $n-1$ will be redundant because shorter paths exist. The diameter of the graph equals $n-1$, which is the shortest distance path between the two most distant nodes on the network.

$$
\begin{cases}
{}^{2}a_{ij} = \sum\limits_{k \mid k\neq i,\ k\neq j} (a_{ik} \times a_{kj}), & i < j \\
{}^{2}a_{ij} = \begin{cases} \dfrac{1}{{}^{2}a_{ji}} & \text{if } {}^{2}a_{ji} > 0 \\ 0 & \text{if } {}^{2}a_{ji} = 0 \end{cases}, & i > j \\
{}^{2}a_{ij} = 1, & i = j
\end{cases}
$$

$$\vdots$$

$$
\begin{cases}
{}^{D}a_{ij} = \sum\limits_{\substack{k,M,\ldots,P \mid \\ k\neq i,\ k\neq j,\ M\neq i, \\ M\neq j,\ldots,P\neq i, \\ P\neq j}} (a_{ik} \times a_{kM} \times \cdots \times a_{Pj}), & i < j \\
{}^{D}a_{ij} = \begin{cases} \dfrac{1}{{}^{D}a_{ji}} & \text{if } {}^{D}a_{ji} > 0 \\ 0 & \text{if } {}^{D}a_{ji} = 0 \end{cases}, & i > j \\
{}^{D}a_{ij} = 1, & i = j
\end{cases}
$$

Further explanation of the problem of redundancies and justification for the solution method adopted is found in Appendix I.

The issue of consistency in a matrix of direct pairwise comparisons can be brought in at this point. We do not require a transitivity condition for the entries of the matrix A_c. In other words, $a_{ij} < 1$ and $a_{jk} < 1$ do not imply $a_{ik} < 1$, nor does $a_{ij} > 1$ and $a_{jk} > 1$ imply $a_{ik} > 1$. This is based on evidence on human judgements reported by Saaty (8).

The vector resulting from total interactions is $T^* = \{t_i\}$, where t_i, the row sum associated with every project i, is a measure of importance (or dominance) of that project with respect to all other projects, for a given criterion. That row sum is defined as follows, where λ

indicates the level of interaction, and ε is a scalar whose function is similar to the function of the scalar s in procedure (b):

$$t_i = \sum_{j=1}^{n} \sum_{\lambda=1}^{D} ({}^{\lambda}a_{ij} \times \varepsilon^{\lambda-1}) - \sum_{\lambda=2}^{D} ({}^{\lambda}a_{ii} \times \varepsilon^{\lambda-1}) .$$

The reason this expression has the form shown is to eliminate redundancies, as explained further in Appendix II.

The difference between the scalar s in procedure (b) and the scalar ε is as follows: in procedure (b) it is assumed that s measures the effectiveness of a direct connection (or comparison), while the scalar ε is a discount factor to be applied to all matrices of pairwise comparisons with powers higher than 1. In other words, procedure (b) assumes that each direct connection has a weight equal to s and all indirect comparisons have decreasing weights. The use of the scalar ε, however, assumes that each direct connection has a weight of 1, while all indirect connections have decreasing weights which are powers of ε.

No major advantage exists between our use of ε and Garrison's. However, we feel that our use of ε is more easily understood than that author's.

Since we do not know the value that ε must assume in a real problem, we run the matrix multiplications and sums, as well as the synthesis, for different values of ε. All results so obtained are displayed for choice.

Given the set of vectors of composite weights, each vector associated to a different ε, the decision maker can then select the project whose sum of composite weights, this sum taken over all values of ε tried, is the highest. The implementation of the framework must

allow the decision maker the generation of more information, that is, the use of additional values of ε, if he so desires.

Once the t_i's have been computed as above, the weight of every project i with respect to a given criterion c can be computed as below:

$$W_{ic} = \frac{t_i}{\sum_{i=1}^{n} t_i} .$$

This number is called the measure of partial performance or the measure of partial utility of project i with respect to criterion c.

To deal with most quantifiable points of view, specific quantitative analysis methodologies such as present value methods are available. For these points of view, it is desirable to assign weights directly to projects in proportion to their performances expressed in natural evaluation units (e.g., net present values of all projects can be determined and weights from 0 to 1 are then proportionally assigned).

Once all vectors of measures of partial utility have been determined, we are able to display such vectors, both for quantifiable and non-quantifiable criteria, in the projects' valuation matrix. We shall now describe how the information contained in this matrix can be synthesized.

III.2 Synthesis of the Projects' Valuation Matrix

Now assume we have determined vectors of measures of partial utility for all quantifiable criteria by utilizing available methodologies and then normalizing measures of performances of all projects. For non-quantifiable criteria, assume we have asked local decision units to fill out A_c matrices, have gone through the computations

described above for each, and have formed a complete projects' valuation matrix including all vectors of measures of partial utility for both quantifiable and non-quantifiable criteria.

Assume also that we have gone to the forest supervisor, have asked him to reveal his tentative preferences with respect to each of the m criteria by forming another A_c matrix, and then have obtained, as it was done for each non-quantifiable criteria, a vector of weights (i.e., now we have one weight for each criterion). Now our problem consists of synthesizing the information contained in the n-by-m projects' valuation matrix and the vector of m weights associated to the m criteria (i.e., now we must obtain a vector of composite project weights that take into account the importance of each project with respect to each criterion as well as the importance of each criterion with respect to all other criteria).

If the vector of m weights associated with the criteria is a column vector, one could think of obtaining a weighted sum of criteria by multiplying the n-by-m projects' valuation matrix by that m-element column vector. However, to do this would be equivalent to assuming the existence of a linear additive utility function as shown below:

$$W_i = \sum_{c=1}^{m} U_c \times W_{ic}$$

where

W_i = overall utility of project i,

U_c = weight of criterion c, and

W_{ic} = utility of project i with respect to criterion c.

It has been shown by many authors, including Fishburn (9), that an additive utility function of the above form exists if and only if

criteria are independent, that is, if the weights of criteria are not interrelated. Since this is not usually the case, a new method must be developed to aggregate weights of criteria and measures of partial utility.

Although one could think of factor-analyzing existing criteria to obtain "new" independent criteria, this method presents conceptual and operational difficulties if the aggregation of criteria is not readily interpretable by both levels of decision making. In other words, losses of information can easily occur because decision makers may find it difficult to compare projects given the combined criteria, or because they may find it difficult to compare mixtures of criteria.

The approach we have adopted to overcome these difficulties follows. We can look at our n investment projects as n points located in a space with m dimensions (i.e., each criterion is associated with one dimension). What we want is an information-preserving method to contract the m-criteria space into a single line.

We define as a reference criterion the one which is valued the most by the forest supervisor, that is, the criterion corresponding to the highest weight in the forest supervisor's vector of weights. However, if the number of criteria seems too large to go through the process of making pairwise comparisons among criteria, the reference criterion can be selected arbitrarily.

If we look at the original matrix of pairwise comparisons of criteria formed by the forest supervisor, we can see that the matrix provides us with trade-off rates between criteria. In particular, if c' is the reference criterion, the row elements corresponding to c' are the trade-off rates between the reference criterion and all other criteria. For example, if c is a particular other criterion, the

trade-off rate between c' and c is the corresponding entry of the matrix, denoted by $a_{c'c}$.

The trade-off rate $a_{c'c}$ can be interpreted as the slope of the forest supervisor's indifference curve for criteria c' and c at a particular point of the curve and at a given point in time. That slope is the marginal rate of substitution on the forest supervisor's utility function and represents the rate at which one criterion can be substituted by another while maintaining a constant utility. This point is further explained in Appendix III.

It must be noted that the trade-off rates among criteria are not necessarily independent of the measures of partial utility, that is, trade-off rates may not be constant. An extension of this research may very well be to investigate the relationship between trade-off rates and measures of partial utility and to incorporate it in the framework.

Now, by assuming that the trade-off rates between criteria are constant, we define a unidimensional space in which the distance between any two projects i and j is defined as follows:

$$\delta(i,j) = \sum_{c=1}^{m} [a_{c'c} \times (W_{ic} - W_{jc})] \ .$$

If $\delta(i,j) > 0$, we can say that project j dominates project i.

We can now form a matrix Δ whose generic element is $\delta(i,j)$. It is, therefore, a matrix of composite distances or indices of relative dominance between projects, and its normalized row sums, each of them denoted by ζ_i, are the composite weights (i.e., measures of overall utility) of each project:

$$\zeta_i = \frac{\sum_{j=1}^{n} \delta(i,j) - \min_i \sum_{j=1}^{n} \delta(i,j)}{\max_i \sum_{j=1}^{n} \delta(i,j) - \min_i \sum_{j=1}^{n} \delta(i,j)} ,$$

Each of those values represents an index of overall dominance of each project, given all other projects as well as quantifiable and non-quantifiable criteria. Through the computation of these weights (or indices) for different values of ε, projects can be ordered and then displayed for choice.

To illustrate the use of the unidimensional distance between projects analytically, assume we have only three criteria, A, B, and C, and that B turns out to be the reference criterion. If this is the case, $\delta(i,j)$ can be computed as follows:

$$\delta(i,j) = a_{BA} \times W_{iA} + 1 \times W_{iB} + a_{BC} \times W_{iC} - (a_{BA} \times W_{jA} + 1 \times W_{jB} + a_{BC} \times W_{jC}) .$$

By summing over all values of j, we have:

$$\sum_{j=1}^{n} \delta(i,j) = \beta_i - C$$

where

$$\beta_i = n \times (a_{BA} \times W_{iA} + 1 \times W_{iB} + a_{BA} \times W_{ic})$$

and C is a constant.

Although this expression for β_i is a linear combination of the measures of partial utility of project i, this approach avoids the problem of dependence among criteria, posed by Fishburn (9). In the above formula, we are dealing with trade-off rates among criteria, and these

rates are independently assigned. Therefore, we are not dealing with an additive utility function whose weights of criteria are interrelated when we use ζ_i as an index of overall utility.

III.3 Example Application

We applied the framework to the selection of a best alternative for an area of the Mt. Baker-Snoqualmie National Forest. The nine alternative projects are presented in Fig. 1.

The problem consisted of determining the best alternative given five evaluation criteria, as follows:

Criterion 1: Minimize discounted costs;

Criterion 2: Minimize adverse impact on soils;

Criterion 3: Minimize adverse impact on watershed;

Criterion 4: Minimize adverse impact on visual aesthetics;

Criterion 5: Minimize adverse impact on traffic safety.

Although in this particular example all criteria consist of minimizing undesirable pay-offs, one could also include positive pay-off evaluations such as relative service to the timber resource, relative service to recreation and public use, etc. The framework can be applied with both types of criteria. The choice of criteria in this example is dictated by the need to compare the results of this method to those obtained in an actual Forest Service study.

The original study was performed in 1973. At that time, the costs of the nine alternative projects were computed using a discount factor of 6%. To evaluate criteria 2 to 5 the following three-reading scale was utilized:

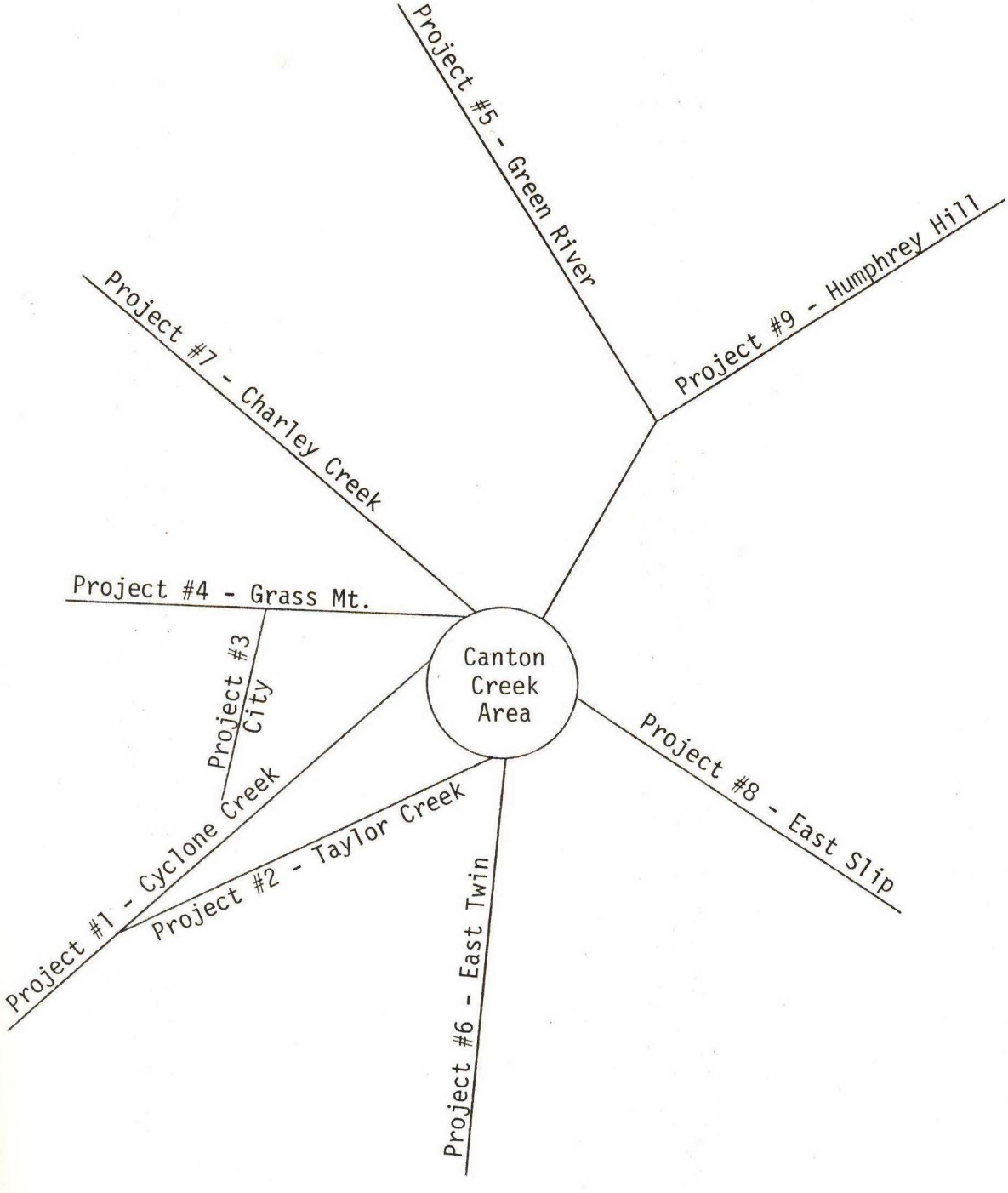

Project #5 - Green River
Project #9 - Humphrey Hill
Project #7 - Charley Creek
Project #4 - Grass Mt.
Project #3 City
Canton Creek Area
Project #8 - East Slip
Project #1 - Cyclone Creek
Project #2 - Taylor Creek
Project #6 - East Twin

Fig. 1 - Example application: the Canton Creek area and the nine alternative projects with their names.

Points	Impact
1	Low
3	Moderate
5	Severe

The assignment of points to each alternative were performed by a team of four specialists, each of them being responsible for one criterion. The results of this assignment are reproduced in Table 1 and the 1973 study is detailed in Ref. 10.

We thought it would be interesting to apply the framework we have developed to this problem and compare results obtained now with those obtained in 1973. Therefore, we asked the transportation planner of the Mt. Baker-Snoqualmie National Forest to fill out five A_c matrices of pairwise comparisons, for the four qualitative criteria (i.e., criteria 2 to 5) as well as for the intercriteria comparisons. The matrices obtained are presented in Tables 2 to 6.

The transportation planner felt that one should not try to compare the four qualitative criteria with Criterion 1. As a consequence, we decided to apply the framework to criteria 2 to 5 only, and then display as inputs for choice both the vectors of composite weights given the simultaneous consideration of these latter four criteria and the vector of normalized discounted costs. In other words, the information displayed for choice will consist of two classes of vectors of weights which are non-quantitatively comparable. One class will consist of different vectors of composite weights, each of them associated with a value of ε. The other class will contain a single vector, that is, the vector of normalized discounted costs.

We chose values of ε equal to $1/k$, where k is an integer varying from 2 to 10. Therefore, giving to the indirect comparisons

Table 1 - Results of the Canton Creek Area Study Performed in 1973

Rating Scale for qualitative criteria: 1 point (low impact), 3 points (moderate impact), 5 points (severe impact)

Ratings were made in their respective areas by Soils Scientist, Bob Snyder; Hydrologist, Bob LaRock; Landscape Architect, Al Grapel; and Transportation Planner, George Larsen.

	Project Number								
	#1	#2	#3	#4	#5	#6	#7	#8	#9
Resource \ Route	Cyclone Creek	Taylor Creek	City	Grass Mountain	Green River	East Twin	Charley Creek	East Slip	Humphrey Hill
Total trans. cost reduced to PNW @ 6% (in $1,000's)*	1,554	1,646	1,683	1,737	1,754	1,828	1,906	1,953	2,392
Cost Rank	Least →								→ Most
Soils	1	3	5	1	1	1	1	3	3
Watershed	1	3	3	1	1	3	3	1	3
Visual Resource	5	1	5	1	1	3	1	3	1
Traffic Safety	3	3	3	1	1	3	1	3	1
Total	10	10	16	4	4	10	6	10	8

*PNW = present net worth

Table 2 - Matrix A_c for Criterion 2

Matrix of pairwise comparisons between projects given the criterion of minimizing adverse impact on soils	Project #1	Project #2	Project #3	Project #4	Project #5	Project #6	Project #7	Project #8	Project #9
Project #1		3	5	$\frac{3}{4}$	1	1	$\frac{3}{4}$	3	2
Project #2			2	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{3}{4}$
Project #3				$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{7}$
Project #4					2	1	1	3	3
Project #5						1	$\frac{1}{2}$	3	1
Project #6							$\frac{3}{4}$	3	2
Project #7								3	3
Project #8									$\frac{1}{2}$
Project #9									

Table 3 - Matrix A_c for Criterion 3

Matrix of pairwise comparisons between projects given the criterion of minimizing adverse impact on watershed	Project #1	Project #2	Project #3	Project #4	Project #5	Project #6	Project #7	Project #8	Project #9
Project #1		3	3	1	1	3	3	1	3
Project #2			1	1/3	1/3	1	1	1/3	1
Project #3				1/3	1/3	1	1	1/3	1
Project #4					1	3	3	1	3
Project #5						3	3	1	3
Project #6							1	1/3	1
Project #7								1/3	1
Project #8									3
Project #9									

Table 4 - Matrix A_c for Criterion 4

Matrix of pairwise comparison between projects given the criterion of minimizing adverse impact on visual aesthetics	Project #1	Project #2	Project #3	Project #4	Project #5	Project #6	Project #7	Project #8	Project #9
Project #1		5	1	1/5	1/5	3/5	1/5	1/2	1/5
Project #2			5	1/3	1/3	1/2	1/4	1/2	1/3
Project #3				1/5	1/5	3/5	1/5	1/2	1/5
Project #4					1	3	1	3	1 1/2
Project #5						3	1	3	1 1/2
Project #6							1/2	1	1/2
Project #7								3	1 1/2
Project #8									1/3
Project #9									

Table 5 - Matrix A_c for Criterion 5

Matrix of pairwise comparisons between projects given the criterion of maximizing traffic safety	Project #1	Project #2	Project #3	Project #4	Project #5	Project #6	Project #7	Project #8	Project #9
Project #1		1	1	1/2	1	1/2	1	1	3/4
Project #2			1	1/2	1	1/2	1	1	3/4
Project #3				1/2	1	1/2	1	1	3/4
Project #4					2	1	1	1 1/2	1
Project #5						1/2	1	1	3/4
Project #6							2	2	1 1/2
Project #7								1	3/4
Project #8									1/2
Project #9									

Table 6 - Matrix A_c for All Criteria Other than Criterion 1

Matrix of intercriteria pairwise comparison	Criteria 2 Minimize adverse impact on soils	Criteria 3 Minimize adverse impact on watershed	Criteria 4 Minimize adverse impact on visual aesthetics	Criteria 5 Maximize traffic safety
Criteria 2		$\frac{3}{4}$	2	$1\frac{3}{4}$
Criteria 3			$2\frac{1}{2}$	2
Criteria 4				$\frac{3}{4}$
Criteria 5				

weights that are multiples of 0.10, 0.11, 0.13, 0.14, 0.17, 0.20, 0.25, 0.33, and 0.50, the alternative projects' valuation matrices presented in Table 7 were obtained. The corresponding vectors of composite priorities are presented in Table 8.

The 1973 study recommended Project #1 as the best alternative. The original assignment of points in this study suggested that either Project #4 or Project #5 had the highest rank based on qualitative criteria. Project #1, however, had the lowest cost. Given that this project was selected as the best, one would normally assume that Criterion 1 was weighted much more than the other four criteria, thus causing the project with the highest weight with respect to Criterion 1, that is, Project #1, to be selected.

Nevertheless, the forest transportation planner feels today that Project #1 has the highest weight with respect not only to Criterion 1, but to all five criteria. On the other hand, when filling out the five A_c matrices of pairwise comparisons, he used as a guideline the weights directly assigned in 1973. Therefore, one should expect, in the present analysis, the following things to happen:

(i) the results obtained now must reflect those obtained in the 1973 study, and

(ii) Project #1 should rank quite high now with respect to its rank in the previous study.

In fact, the present results agree closely with the previous study, which supports the feeling that the algorithm is able to translate the preference structures of the four different analysts into a composite priority order. Besides, the feeling that the relative importance of Project #1 should increase is confirmed in the results from the algorithm, as displayed in Table 8. The fact that the current

Table 7 - Projects' Valuation Matrices for Different Values of ε

Note: Column 1 is the same for all values of ε.

	Investment project number	Criterion number 1	2	3	4	5
$\varepsilon = 0.10$	1	.09	.22	.10	.11	.18
	2	.10	.07	.08	.11	.06
	3	.10	.06	.04	.10	.02
	4	.11	.19	.20	.18	.20
	5	.11	.18	.19	.09	.12
	6	.11	.06	.07	.18	.13
	7	.12	.05	.17	.09	.18
	8	.12	.14	.05	.07	.05
	9	.14	.03	.11	.08	.07
	Investment project number	Criterion number 1	2	3	4	5
$\varepsilon = 0.11$	1	.09	.22	.10	.11	.18
	2	.10	.07	.08	.11	.06
	3	.10	.07	.04	.10	.02
	4	.11	.19	.20	.18	.20
	5	.11	.19	.20	.09	.12
	6	.11	.06	.07	.18	.13
	7	.12	.05	.17	.08	.18
	8	.13	.13	.05	.06	.05
	9	.14	.03	.10	.07	.06
	Investment project number	Criterion number 1	2	3	4	5
$\varepsilon = 0.13$	1	.09	.22	.10	.12	.18
	2	.10	.07	.08	.11	.06
	3	.10	.07	.04	.11	.02
	4	.11	.19	.21	.18	.20
	5	.11	.19	.20	.09	.12
	6	.11	.06	.07	.18	.13
	7	.12	.05	.17	.08	.18
	8	.12	.13	.05	.06	.04
	9	.14	.03	.08	.06	.05

Table 7 (continuation)
Projects' Valuation Matrices for Different Values of ε
Note: Column 1 is the same for all values of ε.

	Investment project number	Criterion number 1	2	3	4	5
$\varepsilon = 0.14$	1	.09	.22	.10	.12	.18
	2	.10	.07	.08	.11	.06
	3	.10	.07	.04	.11	.02
	4	.11	.20	.22	.19	.20
	5	.11	.19	.21	.10	.13
	6	.11	.06	.07	.18	.14
	7	.12	.05	.17	.09	.18
	8	.13	.13	.04	.06	.04
	9	.14	.02	.07	.05	.04
	Investment project number	Criterion number 1	2	3	4	5
$\varepsilon = 0.17$	1	.09	.22	.10	.12	.18
	2	.10	.07	.08	.11	.06
	3	.10	.07	.04	.11	.03
	4	.11	.20	.23	.19	.21
	5	.11	.20	.22	.10	.13
	6	.11	.06	.07	.19	.14
	7	.12	.05	.17	.09	.18
	8	.12	.13	.04	.05	.04
	9	.14	.01	.04	.04	.03
	Investment project number	Criterion number 1	2	3	4	5
$\varepsilon = 0.20$	1	.09	.21	.10	.12	.17
	2	.10	.07	.08	.11	.06
	3	.10	.07	.04	.11	.03
	4	.11	.20	.23	.20	.21
	5	.11	.20	.23	.10	.14
	6	.11	.06	.08	.20	.15
	7	.12	.05	.18	.09	.19
	8	.12	.13	.04	.05	.04
	9	.14	.01	.02	.02	.02

Table 7 (continuation)
Projects' Valuation Matrices for Different Values of ε
Note: Column 1 is the same for all values of ε.

	Investment project number	Criterion number 1	2	3	4	5
ε = 0.25	1	.09	.21	.09	.11	.17
	2	.10	.07	.07	.11	.06
	3	.10	.07	.04	.11	.03
	4	.11	.20	.24	.20	.21
	5	.11	.20	.23	.11	.14
	6	.11	.06	.08	.20	.16
	7	.12	.05	.19	.10	.20
	8	.12	.14	.04	.05	.04
	9	.14	.00	.01	.01	.01
	Investment project number	Criterion number 1	2	3	4	5
ε = 0.33	1	.09	.20	.09	.11	.16
	2	.10	.07	.07	.11	.06
	3	.10	.07	.04	.11	.03
	4	.11	.20	.24	.20	.21
	5	.11	.20	.24	.11	.14
	6	.11	.06	.08	.21	.16
	7	.12	.05	.20	.10	.20
	8	.12	.14	.04	.06	.04
	9	.14	.00	.00	.00	.00
	Investment project number	Criterion number 1	2	3	4	5
ε = 0.50	1	.09	.20	.09	.11	.16
	2	.10	.07	.07	.11	.06
	3	.10	.07	.04	.11	.03
	4	.11	.20	.24	.20	.21
	5	.11	.20	.24	.11	.14
	6	.11	.06	.08	.21	.16
	7	.12	.06	.20	.10	.21
	8	.12	.15	.04	.06	.04
	9	.14	.00	.00	.00	.00

Table 8 - Vectors of Composite Ranks for All Projects and for All Values of ε Tried

Notes: (a) Each rank is a number which indicates the order of a composite weight given the vector of composite weights.
(b) The circles indicate the project that was ranked with first priority.

project number	Vectors of composite priorities								
	$\varepsilon = 0.10$	$\varepsilon = 0.11$	$\varepsilon = 0.13$	$\varepsilon = 0.14$	$\varepsilon = 0.17$	$\varepsilon = 0.20$	$\varepsilon = 0.25$	$\varepsilon = 0.33$	$\varepsilon = 0.50$
1	3	3	3	3	3	4	4	5	5
2	6	6	6	6	6	6	6	6	6
3	9	9	8	7	7	7	7	7	7
4	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)
5	2	2	2	2	2	2	2	2	2
6	5	5	5	5	5	5	5	4	4
7	4	4	4	4	4	3	3	3	3
8	8	8	7	8	8	8	8	8	8
9	7	7	9	9	9	9	9	9	9

procedure is able to distinguish differences among projects relative to all qualitative criteria indicates an improvement over the method used in 1973.

The new procedure is superior to the method used in 1973 or to other similar procedures for the following main reasons:

(a) the use of pairwise comparisons among projects provides a deeper insight into the relative importance of each project than the direct assignment of weights, and

(b) the inclusion of measures of indirect pairwise comparisons among projects in the framework makes evaluation more realistic than if only direct pairwise comparisons are made.

III.4 Chapter Summary

The mathematical development of the basic framework is presented.

We start by showing how a matrix of pairwise comparisons is formed. An algorithm to determine measures of partial utility for non-quantifiable criteria is introduced. The algorithm takes into account total (direct and indirect) pairwise comparisons among projects. Combined with vectors of partial measures of utility for quantifiable criteria, the utilization of the algorithm yields the content of a projects' valuation matrix.

The method of synthesizing information contained in a projects' valuation matrix by taking into account a matrix of comparisons among criteria is presented.

The framework is applied to an actual Forest Service situation. The application suggests that the algorithm provides results consistent with value structures embodied in the pairwise comparisons.

Chapter IV

Heuristic Procedure to Reduce the Number of Comparisons

IV.1 Formation of a Similarity Matrix

In an actual application, too much time would be spent by a decision maker to estimate all entries of a matrix of pairwise comparisons. If 30 projects are being considered, for example, 435 comparisons would have to be made. This is too large to be handled. Therefore, a procedure to reduce the number of pairwise comparisons needed must be developed. The procedure adopted is based on graph and information theory.

Given a set of stimuli i, $i = 1,\ldots,n$, we assume that the greatest amount of information that a typical observer can provide about the stimuli on the basis of judgement is greater when pairwise comparisons are made than when absolute scoring judgements are utilized. Indeed, continuing work in applied psychology supports the thesis that pairwise judgements represent an improvement over absolute scoring judgements (11,12).

Now assume that every project is a stimulus and an observer (i.e., a decision maker) chooses an interval scale from 0 to 7 to assign absolute scores (i.e., readings on the criterion scale) to every stimulus based upon tentative judgements regarding the importance of each stimulus for the given criterion. These tentative absolute judgements must reflect a first-level perception of the importance of each project. Therefore, as compared with the weights determined by considering pairwise comparisons among projects, these tentative absolute judgments represent very rough estimates of the actual importance of each project.

Each tentatively assigned score, therefore, represents the response of an observer to a given stimulus for a certain criterion. We want to develop a method by which these responses can be improved by making pairwise comparisons of stimuli given that only a minimal number of comparisons will be made. This minimal number of comparisons must satisfy the meaningfulness condition of a matrix of pairwise comparisons. In other words, the matrix must correspond to a strongly connected graph.*

For any two stimuli i and j we define a measure of similarity between the two stimuli as follows:

$$S_{ij} = |R_i - R_j|$$

where

R_i = score (or response), based upon absolute judgements, to stimulus i, and

R_j = score (or response), based upon absolute judgements, to stimulus j.

We can say, therefore, that two projects cluster together (or are quite similar), given a certain criterion, if their associated measure of similarity lies below some critical value, S^*.

We are not interested in making pairwise comparisons of projects that cluster together, i.e., we rather look at projects that appear to differ significantly, based upon value judgements. Therefore, in a matrix of pairwise comparisons, we can set the entries corresponding to clustered projects equal to zero. The maximum number of these entries must be such that the resulting non-zero entries, omitting those which

*A strongly connected digraph is defined as a directed graph in which one can go from every node to every other node, without passing by any intermediary node more than once. In other words, given any two nodes i and j, there exists a simple path connecting i to j.

would equal zero in a complete matrix anyway, correspond to a strongly connected graph.

As will be seen through an example, the critical value S^* is perfectly determined, for a given set of projects, considering that we are searching for a minimal strongly connected graph.

A matrix whose elements are the measures of similarity is named the "similarity matrix." It is based upon information contained in this matrix that a search for S^* can be performed.

This method for reducing the number of pairwise comparisons seems to be useful if the number of projects is large, say, $n \geq 10$. If $n = 10$, for example, 45 entries would be needed to form a full matrix of pairwise comparisons.

The method is heuristic in the sense that we reduce the cost of search at the expense of loosing some information. This is, in fact, a characteristic of heuristic search procedures, as pointed out by Harris (13).

The efficiency of the method depends upon how close the readings in the chosen interval scale are. Indeed, our experience with the method indicates that reduction increases when these readings are well differentiated.

There may occur, however, anchoring effects which are likely to reduce the efficiency of the method by concentrating responses to stimuli on the ends of the continuum (14,15). Therefore, it seems that only experience with the method can indicate the best interval scale to be used.

It may also occur that a decision maker will make better pairwise comparisons for fewer entries than for the complete matrix, particularly when the number of projects is very large. As a consequence, one cannot

always judge this reduction method by simply checking how close the vector of composite weights for the reduced matrix is to the vector obtained for the complete matrix.

To analyze the trade-off between the reduction in the number of pairwise comparisons employed and the loss of information implied by working with the reduced matrix, we will henceforth assume that a complete matrix is developed without bias or fatigue effects.

IV.2 Computing Loss of Information

The information content of a matrix of judgements can be computed from the expression:

$$I = \sum_{i=1}^{n} \sum_{j=1}^{n} \left[p_{ij} \times \log \frac{p_{ij}}{p_{i.} \times p_{.j}} \right] \qquad (1)$$

where p_{ij} is interpreted as the probability of a random variable and all probabilities are computed as follows:

$$p_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}} \qquad i, j = 1,\ldots,n$$

$$p_{i.} = \frac{\sum_{j=1}^{n} a_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}} \qquad i = 1,\ldots,n$$

$$p_{.j} = \frac{\sum_{i=1}^{n} a_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}} \qquad j = 1,\ldots,n$$

where the a_{ij}'s are the entries of a matrix of pairwise comparisons.

That measure of information is the same as that proposed by Skolka for an input-output matrix and reproduced by Theil (16).

The relative loss of information implied by working with a reduced matrix can be defined as:

$$\Delta I = \frac{I_c - I_r}{I_c}$$

where

I_c = information content of the complete matrix,

I_r = information content of the reduced matrix,

and both I_c and I_r are computed by Eq. (1).

On the other hand, the relative reduction in the number of pairwise comparisons can be computed as:

$$\Delta N = \frac{N_c - N_r}{N_c}$$

where

N_c = number of comparisons in the complete matrix, and

N_r = number of comparisons in the reduced matrix.

Our limited experience with this procedure suggests that ΔI should be no greater than .30. If the loss of information ΔI is greater than .30, too much information is apparently lost. If this occurs, assuming that more experience with this reduction method shows that the vectors of composite priorities obtained by using the reduced and the complete matrix differ substantially, then it is advisable to avoid using the reduced matrix. On the other hand, given S^*, one can always reduce it a little bit and then obtain a strongly connected graph whose associated matrix will contain more information than that

matrix associated to the minimal strongly connected graph. In other words, the existing trade-off between ΔI and ΔN must be explored in a practical situation.

IV.3 Example Application

We consider, as an example, the criterion of visual aesthetics with data from Mt. Baker-Snoqualmie National Forest. We reproduce below the complete matrix of pairwise comparisons for the 9 projects:

projects	projects								
	1	2	3	4	5	6	7	8	9
1		5	1	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$
2			5	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$
3				$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$
4					1	3	1	3	$1\frac{1}{2}$
5						3	1	3	$1\frac{1}{2}$
6							$\frac{1}{2}$	1	$\frac{1}{2}$
7								3	$1\frac{1}{2}$
8									$\frac{1}{3}$
9									

Tentative absolute judgements regarding the importance of every project are presented below. These values vary from 1 to 5, where 1 indicates low impact, 3 indicates moderate impact, and 5 indicates severe impact.

project	1	2	3	4	5	6	7	8	9
score	5	1	5	1	1	3	1	3	1

The corresponding matrix of similarities is presented below:

projects	1	2	3	4	5	6	7	8	9
1		⑷	0	⑷	⑷	(2)	⑷	(2)	⑷
2			⑷	0	0	(2)	0	(2)	0
3				⑷	⑷	(2)	⑷	(2)	⑷
4					0	(2)	0	(2)	0
5						(2)	0	(2)	0
6							(2)	0	(2)
7								(2)	0
8									(2)
9									

If we set the critical value S^* equal to 4, only the entries surrounding by two circles would receive pairwise comparisons and the resulting graph would not be strongly connected. If we set S^* equal to 2, however, all entries surrounded by at least one circle would receive pairwise comparisons. The resulting graph would then be strongly connected, and it would be a minimal strongly connected graph.

The reduction in the number of pairwise comparisons needed is the following:

$$\Delta N = \frac{36-24}{36} = 33.3\%$$

The reduced matrix is the following, assuming that the values of

pairwise comparisons would be the same as those presented in the complete matrix:

projects	1	2	3	4	5	6	7	8	9
1		5		$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$
2			5			$\frac{1}{2}$		$\frac{1}{2}$	
3				$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$
4						3		3	
5						3		3	
6							$\frac{1}{2}$		$\frac{1}{2}$
7								3	
8									$\frac{1}{3}$
9									

Information contents of both the complete and the reduced matrices were computed according to Eq. (1), and the following results were obtained:

$$I_c = 0.058$$

$$I_r = 0.058$$

The corresponding loss of information for this criterion is therefore zero.

The algorithm for reducing the matrix of pairwise comparisons of projects was applied and measures of partial utility were obtained from the information contained in the reduced matrix.

Corresponding losses of information computed from reduced matrices associated with the other non-quantitative criteria in our example were all below .30. These values were the following:

Criterion 2 - Minimize adverse impact on soils: .21

Criterion 3 - Minimize adverse impact on watershed: .13

Criterion 4 - Minimize adverse impact on visual aesthetics: .18

Criterion 5 - Minimize adverse impact on traffic safety: .09

Vectors of composite weights were computed from the four reduced matrices and for values of ε equal to 0.10, 0.11, 0.13, 0.14, 0.17, 0.20, 0.25, 0.33, and 0.50.

The same alternative that ranked as first priority when the complete matrices were used (i.e., project no. 4) ranked as first priority when reduced matrices were utilized. As a consequence, given that the nine alternatives are mutually exclusive and that the object of the study is to select a single project, differences between ranks were irrelevant for all values of ε tried. We may say, therefore, that the final result was insensitive to a reduction in the number of pairwise comparisons.

It must be noted that such a reduction would have been greater if a four or more reading scale had been used. This scale could be, for example, the following: 1 (very low impact), 3 (low impact), 5 (moderate impact), and 7 (severe impact).

IV.4 Chapter Summary

In an actual application too much time would be spent by a decision maker to estimate all entries of a matrix of pairwise comparisons. To address this problem a heuristic procedure to reduce the number of pairwise comparisons needed is presented in this chapter.

We start by assigning rough absolute scores to all projects for each criterion. These are tentative weights that reflect a first-level perception of the importance of each project. The method proceeds to ignore a certain number of entries of the associated matrix of pairwise comparisons. The number and locations of the entries ignored are such that meaningfulness of the information contained in the reduced matrix is guaranteed. A numerical example is given.

The reduction procedure is therefore very useful in practical applications of the framework in which the number of projects is greater than, say, 10.

Chapter V

Analysis of Contingent Investment Projects

V.1 The Concept of "Contingency Factor"

When, given a set of n projects, there are some which are dependent upon others, the measures of partial utility of these dependent projects, read from the projects' valuation matrix, must be corrected. In other words, for any dependent project i, its corresponding row values in the projects' valuation matrix must be multiplied by a factor that reflects the dependence of project i's measures of partial utility upon at least one other project.

To illustrate this point, assume we have determined the measures of partial utility of i by considering pairwise comparisons only (i.e., by assuming that all contemplated projects are mutually exclusive), and have obtained the following set of measures, given m criteria:

$$\{W_{i1}, W_{i2}, \ldots, W_{im}\} .$$

What we want now is a number K_i, which we call a "contingency factor," such that new measures of partial utility, now reflecting interdependences among projects, are obtained as below:

$$\{K_i \times W_{i1},\ K_i \times W_{i2}, \ldots, K_i \times W_{im}\} .$$

To form those products, in general, the projects' valuation matrix obtained under the assumption that projects are mutually exclusive, must be premultiplied by the following n-by-n diagonal matrix:

$$K = \begin{pmatrix} K_1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & K_2 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & K_i & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & K_n \end{pmatrix}$$

For a project i, the following holds:

$K_i < 1 \Rightarrow$ The value of adopting project i decreases with the adoption of at least one other project,

$K_i = 1 \Rightarrow$ The value of adopting project i is independent of adoption of any other project, and

$K_i > 1 \Rightarrow$ The value of adopting project i increases with the adoption of at least one other project.

Two projects i and j can be related by one of the following relationships:

(i) project i is positively influenced by adoption of j, that is, the adoption of project j contributes to increase the measure of partial utility of project i (i.e., $K_i > 1$);

(ii) project i is negatively influenced by j, that is, the adoption of project j contributes to decrease the measure of partial utility of project i (i.e., $K_i < 1$).

Compound and mutually exclusive projects, respectively, are but extreme cases of (i) and (ii).

In the next sections we describe a method to compute contingency factors for projects.

V.2 The Meaning of Contingency Between Investment Projects

Contingent projects are those projects whose measures of partial

utility change if other projects are adopted; we say that contingent projects have dependent measures of partial utility.

At this point, we must make precise what we mean by a dependent measure of partial utility. Before doing it, we need to stress that we are referring to spatial interrelations only, that is, we ignore the time dimension in our considerations.

Dependence between utilities of any two forest road projects i and j is likely to occur whenever some relevant attributes (e.g., the estimated traffic flows on projects i and j) are interrelated. Interrelations in terms of other attributes such as measures of environmental pollution or measures of traffic safety, for example, can also be taken into account. In any case, we assume it is possible to identify some relevant attribute for any given criterion which can serve as a basis for defining interrelationships among projects. Here, we use traffic flows to illustrate how measures of interrelation can be used.

Although interrelationships in terms of attributes such as traffic flows have been handled by network analysis procedures (17,18), no attempt has so far been made to incorporate these interrelationships into a choice-oriented framework. What these network analysis procedures have attempted to do is to optimize certain criteria by considering interrelationships at the level of attributes such as traffic flows as well as structural investment constraints, that is, budget costs, outputs, manpower, etc. Indeed, no attempt is made to include different evaluation viewpoints into the analysis. It is in this sense that we can say that network analysis procedures do not treat evaluation as a publicly accountable process. Although choice-oriented rating schemes have been developed, those have been more suitable for treating mutually exclusive objects, either plans or projects (19,20,21).

Interrelationships in terms of traffic flows, however, are not generally related to pairwise comparisons of projects (or stimuli), because those comparisons take into account attributes which are not causally related to traffic flows. What we need, therefore, is to suggest a method by which measures of partial utility of projects will reflect interrelationships in terms of traffic flows as well as pairwise comparisons of projects.

However, we do not attempt to convert all interrelationships that exist in terms of traffic flows into contingency factors to be applied to all projects. Rather, we assume that, if two projects are too far apart, their measures of partial utility are independent. As a practical guide, we suggest that any given project i is considered potentially dependent upon another project j if at least 30% of the traffic on i is also on j.

If any two projects are independent in terms of traffic flows, then pairwise comparisons assuming mutual exclusivity are necessary and sufficient to estimate relative utilities. If the two projects are interdependent, however, pairwise comparisons are necessary but not sufficient, and contingency factors must be utilized.

In other words, given two projects i and j that are independent in terms of traffic flows, the ratio between their measures of partial utility can be obtained by simply dividing their composite weights read from the vector produced by making pairwise comparisons. If i and j are interdependent, however, the ratios between their utilities can be determined by using both a contingency factor and the algorithm based on pairwise comparisons; the contingency factor being used to correct the measures of partial utility computed by making the (false) assumption that i and j are mutually exclusive.

As is shown below, this provides us with means to quantify a minimal causal structure embedded in a set of interdependent projects.

As an example, we consider two projects i and j, and assume that they are contingent, i.e., the worth of i depends upon the adoption of j. If W_j measures the utility of j with respect to a particular criterion, and $W_{i/j}$ measures the utility of i with respect to the same criterion given that j is adopted, we can say that the following causal relations exist in terms of measures of partial utilities and traffic flows, where a single arrow represents influence, and a double arrow is used as a substitute for the word "implies":

$$T_j \longrightarrow T_{i/j} \Longrightarrow \text{project i depends on project j}$$

where

T_j = traffic flow on project j, and

$T_{i/j}$ = traffic flow on project i given that project j is adopted.

Given this concept of minimal causality, it will be seen now how we can make it operational.

By the partial utility of a project with respect to a given criterion, we understand, following Stevens (22), the typical response of a median observer in a population of interest to the stimulus represented by the various attributes of the project. According to Fechner's law, as presented by Thurstone (23), utilities can be related to stimuli by the following equation:

$$W = \rho \times \log S$$

where

W = utility,

S = intensity of the stimulus, and

ρ = a constant.

By differentiating the above expression, we obtain

$$\frac{dW}{dS} = \rho \times \frac{1}{S}$$

which suggests that the project marginal utility is a decreasing function of the intensity of the stimulus.

In this case in which the utility of a contemplated project depends upon other projects in a way represented by their independence in terms of traffic flows, we can say that the intensity of the stimuli giving rise to utility can be represented as being proportional to traffic flows. Therefore, it is reasonable to assume that the marginal utility of a project decreases as its traffic flow increases, for all criteria. We assume, as a consequence, Fechner's law to hold for all projects under the condition that ρ is fixed for a given project.

Accordingly, we can measure the degree to which any two projects i and j are interdependent by the following ratio:

$$\frac{W_{i/\sim j}}{W_{i/j}} = \frac{\log T_{i/\sim j}}{\log T_{i/j}} = \eta_{ij} \qquad (2)$$

where

$W_{i/\sim j}$ = utility of i given that j is not adopted, i.e., assuming mutual exclusivity, and

$W_{i/j}$ = utility of i given that j is adopted.

In Eq. (2), if $T_{i/\sim j} = 0$, we observe that project i is, in fact, a continuation of project j. In this limiting case of contingency, i and j should be treated as a single project, rather than as two

distinct projects.

By looking at Eq. (2) and taking into account the assumption that $T_{i/\sim j}$ equals $T_{i/j}$ if projects i and j are independent, we can see that η_{ij} then equals 1.

Eq. (2) and elementary probability concepts will now be utilized to develop a method by which contingency factors can be derived for different actual situations.

V.3 A Method to Determine Contingency Factors

The word "hierarchy" is used here to indicate a level of importance or dominance in a network. For example, in a network with two hierarchies only, two mutually exclusive sets of projects can be identified: one set contains all projects which are dominated by at least one other project which is contained in the other set. In this case, the former set characterizes the lowest hierarchy, and the latter set is associated with the highest hierarchy.

We will present methods to determine contingency factors for the following situations:

--Two hierarchies exist, and one project is dependent upon one other project only.

--Two hierarchies exist, but one project may depend upon two other projects.

--The extent to which one project depends on another project is dependent on a third project.

Although many other cases of dependence can occur in a real road network, other cases can be handled in similar ways. In other words, the same approach we develop to deal with the three cases mentioned above can be applied to different cases of dependence.

Case 1: Two hierarchies exist, and one project is dependent upon one other project only.

Assume we have two projects i and j, and that project i is influenced by project j. We define the following:

P_j = estimated probability that project j is executed,

$W_{i/j}$ = utility of project i given that project j is executed,

$W_{i/\sim j}$ = utility of project i given that project j is not executed, and

W_i = utility of project i with respect to the given criterion, interpreted as the likelihood that project i satisfies the criterion.

We can write the equation below:

$$W_i = W_{i/j} \times P_j + W_{i/\sim j} \times (1-P_j)$$

and, since $K_i = \frac{W_i}{W_{i/\sim j}}$ and $\eta_{ij} = \frac{W_{i/\sim j}}{W_{i/j}}$, we have:

$$K_i = (\frac{1}{\eta_{ij}} - 1) \times P_j + 1 \ . \qquad (3)$$

Note that the probability P_j depends upon K_i, that is, the probability that project j is executed depends upon that project's composite weight, after its correction using the contingency factor K_i. However, that probability also depends upon the acceptance of that project by groups involved in the decision making process other than Forest Service technical analysts. Since the forest supervisor is the coordinator of the process, therefore being most able to understand participants' reactions, it is reasonable to expect that he is the best individual to estimate P_j. As a consequence, we suggest that the

forest supervisor be asked to provide an a priori estimate for P_j; the contingency factor K_i can then be computed accordingly.

Case 2: Two hierarchies exist, but one project depends upon two other projects.

Assume that project i directly depends upon projects j and 1. The following can then be defined:

$W_{i/j\sim 1}$ = utility of project i given that project j is adopted and project 1 is not adopted;

$P_{j\sim 1}$ = estimated probability that project j is adopted and project 1 is not adopted;

$W_{i/j1}$ = utility of project i given that projects j and 1 are adopted;

P_{j1} = estimated probability that projects j and 1 are adopted;

$W_{i/\sim j1}$ = utility of project i given that project j is not adopted and that project 1 is adopted;

$P_{\sim j1}$ = estimated probability that project j is not adopted and that project 1 is adopted;

$W_{i/\sim j\sim 1}$ = utility of project i given that both projects j and 1 are not adopted;

$P_{\sim j\sim 1}$ = estimated probability that both projects j and 1 are not adopted;

W_i = utility of project i with respect to the given criterion, that is, likelihood that project i satisfies the criterion.

Therefore, the following can be written:

$$W_i = W_{i/j\sim 1} \times P_{j\sim 1} + W_{i/j1} \times P_{j1} + W_{i/\sim j1} \times P_{\sim j1} + W_{i/\sim j\sim 1} \times P_{\sim j\sim 1} .$$

By dividing the above equation by $W_{i/\sim j\sim 1}$, we obtain the following:

$$K_i = \frac{W_i}{W_{i/\sim j\sim l}} = \frac{1}{\eta'_{ijl}} \times P_{j\sim l} + \frac{1}{\eta''_{ijl}} \times P_{jl} + \frac{1}{\eta'''_{ijl}} \times P_{\sim jl} + P_{\sim j\sim l} \quad (4)$$

where

$$\eta'_{ijl} = \frac{\log T_{i/\sim j\sim l}}{\log T_{i/j\sim l}},$$

$$\eta''_{ijl} = \frac{\log T_{i/\sim j\sim l}}{\log T_{i/jl}}, \text{ and}$$

$$\eta'''_{ijl} = \frac{\log T_{i/\sim j\sim l}}{\log T_{i/\sim jl}}.$$

As in Case 1, once η'_{ijl}, η''_{ijl}, and η'''_{ijl} are determined, we rely on the forest supervisor's a priori estimates of the probabilities $P_{j\sim l}$, P_{jl}, $P_{\sim jl}$, and $P_{\sim j\sim l}$.* Once these are provided, Eq. (4) can be used to compute the contingency factor K_i.

Case 3: The extent to which one project depends on a second project is dependent upon a third project.

Assume that project i directly depends on project j, and that the extent to which i is influenced by j depends upon a third project l. In this case, we define the following:

$W_{i/jl}$ = utility of project i given that both projects j and l are adopted;

P_{jl} = estimated probability that both projects j and l are adopted;

$W_{i/j\sim l}$ = utility of project i given that project j is adopted and project l is not adopted;

$P_{j\sim l}$ = estimated probability that project j is adopted and project l is not adopted;

$W_{i/\sim j}$ = utility of project i given that project j is not adopted;

*These four probabilities must add up to 1.

$P_{\sim j}$ = estimate probability that project j is not adopted;

W_i = utility of project i with respect to the given criterion, that is, likelihood that project i satisfies the criterion.

We have:

$$W_i = W_{i/jl} \times P_{jl} + W_{i/j\sim l} \times P_{j\sim l} + W_{i/\sim j} \times P_{\sim j} .$$

After dividing the above equation by $W_{i/\sim j\sim l}$, we obtain the following:

$$K_i = \frac{W_i}{W_{i/\sim j\sim l}} = \frac{1}{\eta^{*}_{ijl}} \times P_{jl} + \frac{1}{\eta^{**}_{ijl}} \times P_{j\sim l} + \frac{1}{\eta^{***}_{ijl}} \times P_{\sim j} \qquad (5)$$

where

$$\eta^{*}_{ijl} = \frac{\log T_{i/\sim j\sim l}}{\log T_{i/jl}} ,$$

$$\eta^{**}_{ijl} = \frac{\log T_{i/\sim j\sim l}}{\log T_{i/j\sim l}} , \quad \text{and}$$

$$\eta^{***}_{ijl} = \frac{\log T_{i/\sim j\sim l}}{\log T_{i/\sim j}} .$$

As in the previous two cases, the numbers η^{*}_{ijl}, η^{**}_{ijl}, and η^{***}_{ijl} must be determined and the probabilities P_{jl}, $P_{j\sim l}$, and $P_{\sim j}$, which add up to 1, estimated by the forest supervisor.

V.4 Example Application

As an example of how the method described in this chapter can be used, assume we have two road links A and B, as shown in Fig. 2, and that projects i and j are defined as follows:

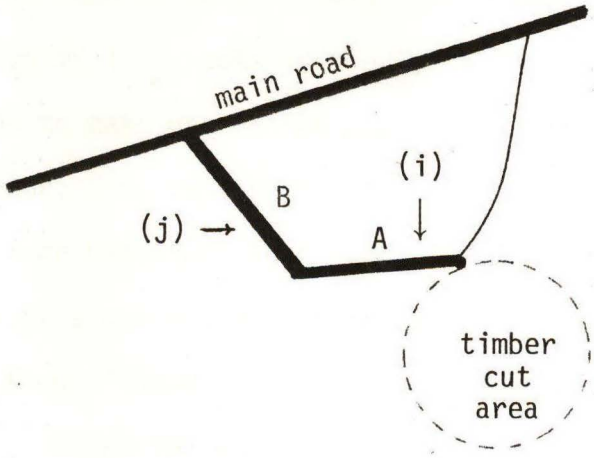


Fig. 2. Example Application

(a) project i is the gravelling of link A, and

(b) project j is the paving of link B.

Assume that road A is tributary to road B, and that the execution of project j is likely to attract more traffic to link B. Since some of this same attracted volume will be using link A, we can say that project i is influenced by project j. If $T_{i/j}$ and $T_{i/\sim j}$ are estimated as 500 A.D.T. and 150 A.D.T., respectively, we have:*

$$\eta_{ij} = \frac{\log 150}{\log 500} = .81 \qquad K_i = (\frac{1}{.81} - 1) \times P_j + 1 = .23 \times P_j + 1 \ .$$

To compute K_i, the forest supervisor is asked to estimate the probability that project j will be executed, that is, P_j. If, based upon his experience as well as his perception of the process, he estimates that project j has a 60% chance of being executed, we have:

$$K_i = .23 \times 0.60 + 1 = 1.14 \ .$$

Using this contingency factor, corrected measures of partial

*A.D.T. = average daily traffic

utility are then computed and normalized.

As is seen in this example, the method suggested to handle contingent investment projects is simple to apply and has the positive feature of trying to make use of the experience and perception of the process coordinator, that is, the forest supervisor, rather than attempting to rely on some procedure whose dependence on judgement may be limited. Although this was a simple example, in a real situation the implementation of our framework as a conversational computer program would be essential to provide direct interaction between a forest supervisor and the framework, so that forest supervisors will be able to select probabilities or ranges for probabilities conveniently.

V.5 Chapter Summary

The framework is extended to analyze situations in which not all projects are mutually exclusive. The extension comprises the computation of contingency factors to correct for interdependencies among projects. These factors are computed and then applied to the projects' valuation matrix obtained by assuming that projects are mutually exclusive.

A method to determine contingency factors is described. The method makes use of elementary probability concepts as well as the decision maker's experience and perception of the process. Formulas to compute contingency factors are presented for cases which are likely to occur in practice; formulas for other cases that may occur can be developed in similar ways. An example application is given. Although interrelationships discussed in this chapter were related to traffic flows, other attributes can be used, such as environmental impacts and traffic safety.

With the introduction of the notion of contingency factors into the framework, it can be applied to situations in which either all projects are mutually exclusive or projects are interdependent.

Chapter VI

Conclusions and Directions for Further Research

It must be noted that an important distinction exists between the planning philosophies behind the described framework and the linear programming approach to multiple objective decision making. As it has been pointed out by MacCrimmon (24), the linear programming approach is more appropriate when it is desired to treat the planning of investments as a design problem rather than a choice problem, since the purpose then is to design an optimal alternative by putting together the best combination of attribute values. The intention behind the framework described here is to present alternatives to be chosen through a political process. It is therefore convenient to use a procedure by which alternatives can be ranked and this rank can be changed through an interactive process involving technical analysts, decision makers, and the public. Such a procedure is explicit within the described framework.

On the other hand, the framework is essentially different from the DELPHI method, in which different structures of order are progressively modified until a unique structure is reached. The DELPHI method is designed to elicit opinions from a group of experts with the aim of formulating judgements for subject matter where precise information is lacking. Thus, it is a method oriented towards the achievement of consensus by revising feedbacks derived from the systematic solicitation and collation of informed judgements on the particular topic under consideration. In our framework, however, we are less concerned with the process by which consensus can be achieved than with the process by which information can be organized and displayed for choice.

Therefore, both the DELPHI method and our framework can be used in conjunction to achieve consensus at the final selection.

An extension of this research should include implementation of a conversational system for information management to include all programs, data files, and summary files needed to allow on-line utilization of the framework by decision makers and technical analysts. This framework will thereby become a man-computer interactive evaluation tool useful to transportation analysts.

An extension of the theory behind the framework should include an investigation of the relationship between trade-off rates and the associate measures of partial utility. As we pointed out in Chapter III, we assumed these trade-off rates to be constant.

The framework is unique in the sense that multiple criteria are dealt with within the context of a choice-oriented framework, and total (direct and indirect) interactions among projects are included in the analysis. Thus, by making evaluation a publicly accountable process, criteria and projects can be introduced or removed in the political arena, according to the opinions of different participants in the process.

Although additional work should be performed to acquire further experience with it, the framework is complete for implementation and application to real planning problems.* It can serve as a basis for increasing the overall effectiveness of the forest road investment decision making process by improving the rationale for combining engineering and resource management criteria as well as by facilitating the openness of the process to public participation.

*A part of this additional work needed consists of comparing, for a real situation, the vector of composite priorities obtained by using, in the projects' valuation matrix, rough absolute scores, with that obtained by making pairwise comparisons.

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Appendix I

Justification of the Solution Method Adopted

By taking successive powers of the matrix A_c, we obtain a finite series of matrices $A_c^2, A_c^3, \ldots, A_c^D$. The entries of those matrices reflect the indirect interaction between two projects.

To make this point clear, the concept of interaction can be explained in graph theoretical terminology. We define the length of a path on a graph as the number of arcs that belong to the path. In general, an ϵ-path is any path comprising ϵ arcs. We also define the intensity of a path of length ϵ from node i to node j as the product of the intensities of the arcs that belong to the path. The total intensity of all ϵ-paths from node i to node j is the sum of the intensities of each path.

Therefore, the total intensity of all ϵ-paths from node i to node j is given by the corresponding element of the matrix A_c^{ϵ}. Each entry ${}^{\epsilon}a_{ij}$ of this generic matrix can be interpreted as the dominance, over all ϵ-paths, exerted by node i over node j.

However, when computing any entry of a generic matrix, we do not want to include redundant paths, that is, paths that pass by either the origin or the destination node more than once. Therefore, when we form the summation expression to compute the upper triangular part of the matrix, we must exclude the possibility of having loops. This explains the mathematical formulation of a generic entry for the case in which i is less than j, assuming that i is the origin node and j is the destination node. That formulation, for a generic ϵ and n projects, is the following:

$$^{\epsilon}a_{ij} = \sum_{\substack{k,M,\ldots,P \mid \\ k\neq i,\, k\neq j,\, M\neq i,\\ M\neq j,\ldots,P\neq i,\\ P\neq j}} \overbrace{(a_{ik} \times a_{kM} \times \cdots \times a_{Pj})}^{\epsilon \text{ factors}} , \qquad i < j .$$

For example, the matrix A_c^2 describes a graph whose path has a length of 2. Given two projects i and j, the sum of all 2-step paths from i to j represents the intensity of all possible indirect interactions of length equal to 2. If $^2a_{ij}$ denotes a generic element of A_c^2, we have:

$$^2a_{ij} = \sum_{k=1}^{n} (a_{ik} \times a_{kj}) , \qquad k \neq i, \quad k \neq j$$

where

a_{ik} = a measure of the relative importance of project i as compared with project k, for a given criterion,

and a_{kj} = a measure of the relative importance of project k as compared with project j, for the given criterion.

Appendix II

Elimination of Redundancies

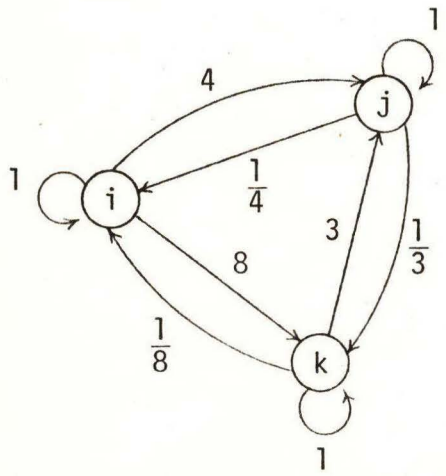
Consider the expression below for computing t_i:

$$t_i = \underbrace{\sum_{j=1}^{n} \sum_{\lambda=1}^{D} ({}^{\lambda}a_{ij} \times \varepsilon^{\lambda-1})}_{\alpha} - \underbrace{\sum_{\lambda=2}^{D} ({}^{\lambda}a_{ii} \times \varepsilon^{\lambda-1})}_{\beta} .$$

The term β is subtracted because the values ${}^{\lambda}a_{ii}$ are all equal to 1, that is, there is a loop with intensity of 1 at each node. Although those values are discounted by multiplying them by $\varepsilon^{\lambda-1}$, we only want to consider the loops once. Therefore, the term β must include all (discounted) values of ${}^{\lambda}a_{ii}$ starting with $\lambda = 2$.

The term α is simply the sum of all (discounted) values of ${}^{\lambda}a_{ij}$, this sum being taken along all columns j. In this term, we include the existing loop with intensity of 1 at the node i. The importance of this loop, however, is abated through the use of discounted factors $\varepsilon^{\lambda-1}$.

As an example, consider the graph below. The matrix of pairwise comparisons A_c as well as the computations to determine t_i are presented for $\varepsilon = .30$.



$$A_c = \begin{bmatrix} a_{ii} & a_{ij} & a_{ik} \\ a_{ji} & a_{jj} & a_{jk} \\ a_{ki} & a_{kj} & a_{kk} \end{bmatrix} = \begin{bmatrix} 1 & 4 & 8 \\ \frac{1}{4} & 1 & \frac{1}{3} \\ \frac{1}{8} & 3 & 1 \end{bmatrix} ;$$

$$D = 3 - 1 = 2 ;$$

$$A_c^2 = \begin{bmatrix} 1 & 24 & \frac{4}{3} \\ \frac{1}{24} & 1 & 2 \\ \frac{3}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

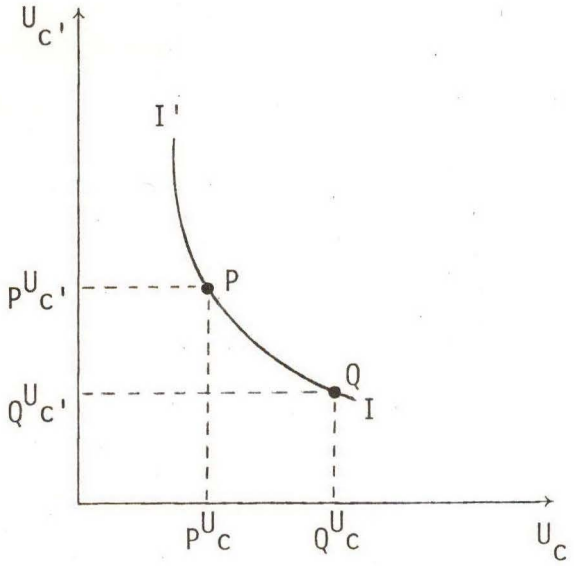
$$t_i = (1+4+8) + (1 \times 0.30 + 24 \times 0.20 + \frac{4}{3} \times 0.30) - 1 \times 0.30$$

$$= \underline{\underline{20.6}}$$

Appendix III

Interpretation of the Trade-Off Rates Among Criteria

The figure shows a decision maker's indifference curve II'. In other words, given that curve, the supervisor is indifferent between any two points P $({}_{P}U_{c'}, {}_{P}U_{c})$ and Q $({}_{Q}U_{c'}, {}_{Q}U_{c})$, i.e., he is indifferent between working with the pair of partial utilities $({}_{P}U_{c'}, {}_{P}U_{c})$ or the pair $({}_{Q}U_{c'}, {}_{Q}U_{c})$, where:



$_{P}U_{c'}$ = utility of criterion c' at point P,

$_{P}U_{c}$ = utility of criterion c at point P,

$_{Q}U_{c'}$ = utility of criterion c' at point Q, and

$_{Q}U_{c}$ = utility of criterion c at point Q.

We can also say that, for any point on II', the overall utility (or level of satisfaction) achieved by the forest supervisor is the same.

If U designates the overall utility achieved by the supervisor, we have, by definition, the following:

$$MRS \equiv -\frac{\frac{\partial U}{\partial U_{c}}}{\frac{\partial U}{\partial U_{c'}}} = -\frac{\partial U_{c'}}{\partial U_{c}}$$

where MRS is the marginal rate of substitution for utility, and

represents the rate at which one criterion can be substituted by another while maintaining a constant overall utility or level of satisfaction for the forest supervisor.

We want to show that the slope of the curve II' is equal to the marginal rate of substitution for utility.

Assume we have the expression for the utility achieved by the forest supervisor as below:

$$U(U_{c'}, U_c) = K \ .$$

By differentiation, we obtain the following:

$$\frac{\partial U}{\partial U_{c'}} \times dU_{c'} + \frac{\partial U}{\partial U_c} \times dU_c = 0 \ ,$$

therefore

$$\frac{\partial U}{\partial U_{c'}} \times dU_{c'} = - \frac{\partial U}{\partial U_c} \times dU_c$$

or, after dividing both terms by ∂U:

$$\underbrace{\frac{dU_{c'}}{dU_c}}_{\text{slope}} = - \underbrace{\frac{\partial U_{c'}}{\partial U_c}}_{\text{MRS}} \ .$$

It must be noted that no assumption is needed on the nature of the function U, except that it must be continuous and differentiable.